

ELLIPTIC INTEGRAL OF THE FIRST KIND $F(\varphi|\alpha)$

Table 17.5

$$F(\varphi|\alpha) = \int_0^\varphi (1 - \sin^2 \alpha \sin^2 \theta)^{-1/2} d\theta$$

$\alpha \backslash \varphi$	0°	5°	10°	15°	20°	25°	30°
0°	0	0.08726 646	0.17453 293	0.26179 939	0.34906 585	0.43633 231	0.52359 878
2	0	0.08726 660	0.17453 400	0.26180 298	0.34907 428	0.43634 855	0.52362 636
4	0	0.08726 700	0.17453 721	0.26181 374	0.34909 952	0.43639 719	0.52370 903
6	0	0.08726 767	0.17454 255	0.26183 163	0.34914 148	0.43647 806	0.52384 653
8	0	0.08726 860	0.17454 999	0.26185 656	0.34919 998	0.43659 086	0.52403 839
10	0	0.08726 980	0.17455 949	0.26188 842	0.34927 479	0.43673 518	0.52428 402
12	0	0.08727 124	0.17457 102	0.26192 707	0.34936 558	0.43691 046	0.52458 259
14	0	0.08727 294	0.17458 451	0.26197 234	0.34947 200	0.43711 606	0.52493 314
16	0	0.08727 487	0.17459 991	0.26202 402	0.34959 358	0.43735 119	0.52533 449
18	0	0.08727 703	0.17461 714	0.26208 189	0.34972 983	0.43761 496	0.52578 529
20	0	0.08727 940	0.17463 611	0.26214 568	0.34988 016	0.43790 635	0.52628 399
22	0	0.08728 199	0.17465 675	0.26221 511	0.35004 395	0.43822 422	0.52682 887
24	0	0.08728 477	0.17467 895	0.26228 985	0.35022 048	0.43856 733	0.52741 799
26	0	0.08728 773	0.17470 261	0.26236 958	0.35040 901	0.43893 430	0.52804 924
28	0	0.08729 086	0.17472 762	0.26245 392	0.35060 870	0.43932 365	0.52872 029
30	0	0.08729 413	0.17475 386	0.26254 249	0.35081 868	0.43973 377	0.52942 863
32	0	0.08729 755	0.17478 119	0.26263 487	0.35103 803	0.44016 296	0.53017 153
34	0	0.08730 108	0.17480 950	0.26273 064	0.35126 576	0.44060 939	0.53094 608
36	0	0.08730 472	0.17483 864	0.26282 934	0.35150 083	0.44107 115	0.53174 916
38	0	0.08730 844	0.17486 848	0.26293 052	0.35174 218	0.44154 622	0.53257 745
40	0	0.08731 222	0.17489 887	0.26303 369	0.35198 869	0.44203 247	0.53342 745
42	0	0.08731 606	0.17492 967	0.26313 836	0.35223 920	0.44252 769	0.53429 546
44	0	0.08731 992	0.17496 073	0.26324 404	0.35249 254	0.44302 960	0.53517 761
46	0	0.08732 379	0.17499 189	0.26335 019	0.35274 748	0.44353 584	0.53606 986
48	0	0.08732 765	0.17502 300	0.26345 633	0.35300 280	0.44404 397	0.53696 798
50	0	0.08733 149	0.17505 392	0.26356 191	0.35325 724	0.44455 151	0.53786 765
52	0	0.08733 528	0.17508 448	0.26366 643	0.35350 955	0.44505 593	0.53876 438
54	0	0.08733 901	0.17511 455	0.26376 936	0.35375 845	0.44555 469	0.53965 358
56	0	0.08734 265	0.17514 397	0.26387 020	0.35400 269	0.44604 519	0.54053 059
58	0	0.08734 620	0.17517 260	0.26396 842	0.35424 101	0.44652 487	0.54139 069
60	0	0.08734 962	0.17520 029	0.26406 355	0.35447 217	0.44699 117	0.54222 911
62	0	0.08735 291	0.17522 690	0.26415 509	0.35469 497	0.44744 153	0.54304 111
64	0	0.08735 605	0.17525 232	0.26424 258	0.35490 823	0.44787 348	0.54382 197
66	0	0.08735 902	0.17527 640	0.26432 556	0.35511 081	0.44828 459	0.54456 704
68	0	0.08736 182	0.17529 903	0.26440 362	0.35530 160	0.44867 252	0.54527 182
70	0	0.08736 442	0.17532 010	0.26447 634	0.35547 959	0.44903 502	0.54593 192
72	0	0.08736 681	0.17533 949	0.26454 334	0.35564 377	0.44936 997	0.54654 316
74	0	0.08736 898	0.17535 712	0.26460 428	0.35579 326	0.44967 538	0.54710 162
76	0	0.08737 092	0.17537 289	0.26465 883	0.35592 721	0.44994 944	0.54760 364
78	0	0.08737 262	0.17538 672	0.26470 671	0.35604 488	0.45019 046	0.54804 587
80	0	0.08737 408	0.17539 854	0.26474 766	0.35614 560	0.45039 699	0.54842 535
82	0	0.08737 528	0.17540 830	0.26478 147	0.35622 881	0.45056 775	0.54873 947
84	0	0.08737 622	0.17541 594	0.26480 795	0.35629 402	0.45070 168	0.54898 608
86	0	0.08737 689	0.17542 143	0.26482 697	0.35634 086	0.45079 795	0.54916 348
88	0	0.08737 730	0.17542 473	0.26483 842	0.35636 908	0.45085 596	0.54927 042
90	0	0.08737 744 $\left[\begin{smallmatrix} (-8)3 \\ 3 \end{smallmatrix} \right]$	0.17542 583 $\left[\begin{smallmatrix} (-7)3 \\ 4 \end{smallmatrix} \right]$	0.26484 225 $\left[\begin{smallmatrix} (-6)1 \\ 4 \end{smallmatrix} \right]$	0.35637 851 $\left[\begin{smallmatrix} (-6)2 \\ 5 \end{smallmatrix} \right]$	0.45087 533 $\left[\begin{smallmatrix} (-6)5 \\ 5 \end{smallmatrix} \right]$	0.54930 614 $\left[\begin{smallmatrix} (-6)9 \\ 5 \end{smallmatrix} \right]$
5	0	0.08726 730	0.17453 962	0.26182 180	0.34911 842	0.43643 361	0.52377 095
15	0	0.08727 387	0.17459 198	0.26199 739	0.34953 092	0.43722 998	0.52512 754
25	0	0.08728 623	0.17469 061	0.26232 912	0.35031 330	0.43874 792	0.52772 849
35	0	0.08730 289	0.17482 397	0.26277 965	0.35138 244	0.44083 848	0.53134 425
45	0	0.08732 185	0.17497 630	0.26329 709	0.35261 989	0.44328 233	0.53562 273
55	0	0.08734 084	0.17512 935	0.26382 007	0.35388 123	0.44580 113	0.54009 391
65	0	0.08735 756	0.17526 454	0.26428 466	0.35501 092	0.44808 179	0.54419 926
75	0	0.08736 998	0.17536 525	0.26463 238	0.35586 223	0.44981 645	0.54735 991
85	0	0.08737 659	0.17541 895	0.26481 840	0.35631 976	0.45075 457	0.54908 352

The table can also be used inversely to find $\varphi = \text{am } u$ where $u = F(\varphi|\alpha)$ and so the Jacobian elliptic functions, for example $\text{sn } u = \sin \varphi$, $\text{cn } u = \cos \varphi$, $\text{dn } u = (1 - \sin^2 \alpha \sin^2 \varphi)^{1/2}$. See Examples 7-11. Compiled from K. Pearson, Tables of the complete and incomplete elliptic integrals, Cambridge Univ. Press, Cambridge, England, 1934 (with permission). Known errors have been corrected.

Table 17.5

ELLIPTIC INTEGRAL OF THE FIRST KIND $F(\varphi|\alpha)$

$$F(\varphi|\alpha) = \int_0^\varphi (1 - \sin^2 \alpha \sin^2 \theta)^{-\frac{1}{2}} d\theta$$

$\alpha \backslash \varphi$	35°	40°	45°	50°	55°	60°
0°	0.61086 524	0.69813 170	0.78539 816	0.87266 463	0.95993 109	1.04719 755
2	0.61090 819	0.69819 436	0.78548 509	0.87278 045	0.96008 037	1.04738 465
4	0.61103 691	0.69838 220	0.78574 574	0.87312 784	0.96052 821	1.04794 603
6	0.61125 108	0.69869 484	0.78617 974	0.87370 649	0.96127 450	1.04888 194
8	0.61155 010	0.69913 161	0.78678 644	0.87451 593	0.96231 911	1.05019 278
10	0.61193 318	0.69969 159	0.78756 494	0.87555 545	0.96366 180	1.05187 911
12	0.61239 927	0.70037 358	0.78851 403	0.87682 412	0.96530 224	1.05394 160
14	0.61294 707	0.70117 608	0.78963 221	0.87832 076	0.96723 998	1.05638 099
16	0.61357 504	0.70209 730	0.79091 768	0.88004 389	0.96947 438	1.05919 813
18	0.61428 140	0.70313 511	0.79236 827	0.88199 174	0.97200 462	1.06239 384
20	0.61506 406	0.70428 706	0.79398 143	0.88416 214	0.97482 960	1.06596 891
22	0.61592 071	0.70555 037	0.79575 422	0.88655 254	0.97794 790	1.06992 405
24	0.61684 871	0.70692 183	0.79768 324	0.88915 992	0.98135 773	1.07425 976
26	0.61784 515	0.70839 788	0.79976 461	0.89198 071	0.98505 681	1.07897 628
28	0.61890 682	0.70997 451	0.80199 389	0.89501 076	0.98904 227	1.08407 347
30	0.62003 018	0.71164 728	0.80436 610	0.89824 524	0.99331 059	1.08955 067
32	0.62121 138	0.71341 124	0.80687 558	0.90167 852	0.99785 743	1.09540 656
34	0.62244 622	0.71526 098	0.80951 599	0.90530 415	1.00267 749	1.10163 899
36	0.62373 019	0.71719 052	0.81228 024	0.90911 465	1.00776 438	1.10824 474
38	0.62505 840	0.71919 335	0.81516 039	0.91310 148	1.01311 039	1.11521 933
40	0.62642 563	0.72126 235	0.81814 765	0.91725 487	1.01870 633	1.12255 667
42	0.62782 630	0.72338 982	0.82123 227	0.92156 370	1.02454 127	1.13024 880
44	0.62925 446	0.72556 741	0.82440 346	0.92601 535	1.03060 230	1.13828 546
46	0.63070 385	0.72778 615	0.82764 941	0.93059 558	1.03687 427	1.14665 369
48	0.63216 783	0.73003 640	0.83095 712	0.93528 835	1.04333 948	1.15533 731
50	0.63363 947	0.73230 789	0.83431 247	0.94007 568	1.04997 735	1.16431 637
52	0.63511 150	0.73458 970	0.83770 010	0.94493 756	1.05676 412	1.17356 652
54	0.63657 639	0.73687 028	0.84110 344	0.94985 177	1.06367 248	1.18305 833
56	0.63802 636	0.73913 751	0.84450 468	0.95479 381	1.07067 128	1.19275 650
58	0.63945 343	0.74137 870	0.84788 483	0.95973 682	1.07772 516	1.20261 907
60	0.64084 944	0.74358 071	0.85122 375	0.96465 156	1.08479 434	1.21259 661
62	0.64220 613	0.74572 998	0.85450 024	0.96950 647	1.09183 436	1.22263 139
64	0.64351 521	0.74781 266	0.85769 220	0.97426 773	1.09879 601	1.23265 660
66	0.64476 839	0.74981 471	0.86077 677	0.97889 946	1.10562 535	1.24259 576
68	0.64595 751	0.75172 208	0.86373 057	0.98336 406	1.11226 392	1.25236 238
70	0.64707 458	0.75352 078	0.86652 996	0.98762 253	1.11864 920	1.26185 988
72	0.64811 189	0.75519 716	0.86915 135	0.99163 507	1.12471 530	1.27098 218
74	0.64906 209	0.75673 800	0.87157 159	0.99536 166	1.13039 401	1.27961 482
76	0.64991 829	0.75813 076	0.87376 830	0.99876 287	1.13561 610	1.28763 696
78	0.65067 415	0.75936 376	0.87572 037	1.00180 067	1.14031 304	1.29492 436
80	0.65132 394	0.76042 640	0.87740 833	1.00443 942	1.14441 892	1.30135 321
82	0.65186 270	0.76130 931	0.87881 481	1.00664 678	1.14787 262	1.30680 495
84	0.65228 622	0.76200 457	0.87992 495	1.00839 470	1.15062 010	1.31117 166
86	0.65259 116	0.76250 582	0.88072 675	1.00966 028	1.15261 652	1.31436 170
88	0.65277 510	0.76280 846	0.88121 143	1.01042 658	1.15382 828	1.31630 510
90	0.65283 658	0.76290 965	0.88137 359	1.01068 319	1.15423 455	1.31695 790
	$\left[\begin{smallmatrix} (-5)2 \\ 5 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-5)3 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-5)4 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-5)6 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)1 \\ 7 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)2 \\ 7 \end{smallmatrix} \right]$
5	0.61113 335	0.69852 295	0.78594 111	0.87338 828	0.96086 405	1.04836 715
15	0.61325 114	0.70162 198	0.79025 416	0.87915 412	0.96832 014	1.05774 229
25	0.61733 857	0.70764 702	0.79870 514	0.89054 388	0.98317 128	1.07657 042
35	0.62308 236	0.71621 617	0.81088 311	0.90718 679	1.00518 803	1.10489 545
45	0.62997 691	0.72667 222	0.82601 788	0.92829 036	1.03371 296	1.14242 906
55	0.63730 374	0.73800 634	0.84280 548	0.95232 094	1.06716 268	1.18788 407
65	0.64414 930	0.74882 464	0.85924 936	0.97660 210	1.10223 077	1.23764 210
75	0.64950 235	0.75745 364	0.87269 924	0.99710 535	1.13306 645	1.28370 993
85	0.65245 368	0.76227 978	0.88036 502	1.00908 899	1.15171 457	1.31291 870

ELLIPTIC INTEGRAL OF THE FIRST KIND $F(\varphi|\alpha)$

Table 17.5

$$F(\varphi|\alpha) = \int_0^\varphi (1 - \sin^2 \alpha \sin^2 \theta)^{-\frac{1}{2}} d\theta$$

$\alpha \backslash \varphi$	65°	70°	75°	80°	85°	90°
0°	1.13446 401	1.22173 048	1.30899 694	1.39626 340	1.48352 986	1.57079 633
2	1.13469 294	1.22200 477	1.30931 959	1.39663 672	1.48395 543	1.57127 495
4	1.13537 994	1.22282 810	1.31028 822	1.39775 763	1.48523 342	1.57271 244
6	1.13652 576	1.22420 180	1.31190 491	1.39962 909	1.48736 769	1.57511 361
8	1.13813 158	1.22612 810	1.31417 314	1.40225 598	1.49036 470	1.57848 658
10	1.14019 906	1.22861 010	1.31709 778	1.40564 522	1.49423 361	1.58284 280
12	1.14273 032	1.23165 180	1.32068 514	1.40980 577	1.49898 627	1.58819 721
14	1.14572 789	1.23525 808	1.32494 296	1.41474 871	1.50463 742	1.59456 834
16	1.14919 471	1.23943 470	1.32988 047	1.42048 728	1.51120 474	1.60197 853
18	1.15313 409	1.24418 827	1.33550 840	1.42703 700	1.51870 904	1.61045 415
20	1.15754 967	1.24952 627	1.34183 901	1.43441 578	1.52717 445	1.62002 590
22	1.16244 535	1.25545 700	1.34888 616	1.44264 399	1.53662 865	1.63072 910
24	1.16782 525	1.26198 957	1.35666 531	1.45174 466	1.54710 309	1.64260 414
26	1.17369 362	1.26913 385	1.36519 359	1.46174 360	1.55863 334	1.65569 693
28	1.18005 472	1.27690 045	1.37448 981	1.47266 958	1.57125 942	1.67005 943
30	1.18691 274	1.28530 059	1.38457 455	1.48455 455	1.58502 624	1.68575 035
32	1.19427 162	1.29434 605	1.39547 013	1.49743 384	1.59998 406	1.70283 594
34	1.20213 489	1.30404 906	1.40720 064	1.51134 644	1.61618 906	1.72139 083
36	1.21050 542	1.31442 210	1.41979 198	1.52633 523	1.63370 398	1.74149 923
38	1.21938 520	1.32547 772	1.43327 179	1.54244 734	1.65259 894	1.76325 618
40	1.22877 499	1.33722 824	1.44766 938	1.55973 441	1.67295 226	1.78676 913
42	1.23867 392	1.34968 545	1.46301 565	1.57825 301	1.69485 156	1.81215 985
44	1.24907 904	1.36286 013	1.47934 287	1.59806 493	1.71839 498	1.83956 672
46	1.25998 475	1.37676 148	1.49668 437	1.61923 762	1.74369 264	1.86914 755
48	1.27138 210	1.39139 640	1.51507 416	1.64184 453	1.77086 836	1.90108 303
50	1.28325 798	1.40676 855	1.53454 619	1.66596 542	1.80006 176	1.93558 110
52	1.29559 414	1.42287 717	1.55513 354	1.69168 665	1.83143 068	1.97288 227
54	1.30836 604	1.43971 560	1.57686 709	1.71910 125	1.86515 414	2.01326 657
56	1.32154 149	1.45726 935	1.59977 378	1.74830 880	1.90143 591	2.05706 232
58	1.33507 910	1.47551 372	1.62387 409	1.77941 482	1.94050 873	2.10465 766
60	1.34892 643	1.49441 087	1.64917 867	1.81252 953	1.98263 957	2.15651 565
62	1.36301 803	1.51390 609	1.67568 359	1.84776 547	2.02813 570	2.21319 470
64	1.37727 323	1.53392 332	1.70336 398	1.88523 335	2.07735 219	2.27537 643
66	1.39159 384	1.55435 972	1.73216 516	1.92503 509	2.13070 052	2.34390 472
68	1.40586 195	1.57507 940	1.76199 085	1.96725 237	2.18865 839	2.41984 165
70	1.41993 796	1.59590 624	1.79268 736	2.01192 798	2.25177 995	2.50455 008
72	1.43365 925	1.61661 644	1.82402 292	2.05903 582	2.32070 416	2.59981 973
74	1.44684 001	1.63693 134	1.85566 175	2.10843 282	2.39615 610	2.70806 762
76	1.45927 266	1.65651 218	1.88713 308	2.15978 295	2.47892 739	2.83267 258
78	1.47073 163	1.67495 873	1.91779 814	2.21243 977	2.56980 281	2.97856 895
80	1.48098 006	1.69181 489	1.94682 231	2.26527 326	2.66935 045	3.15338 525
82	1.48977 975	1.70658 456	1.97316 666	2.31643 897	2.77736 748	3.36986 803
84	1.49690 410	1.71876 033	1.99562 118	2.36313 736	2.89146 664	3.65185 597
86	1.50215 336	1.72786 543	2.01290 452	2.40153 358	3.00370 926	4.05275 817
88	1.50537 033	1.73350 464	2.02384 126	2.42718 003	3.09448 898	4.74271 727
90	1.50645 424 [$\begin{smallmatrix} (-4)3 \\ 8 \end{smallmatrix}$]	1.73541 516 [$\begin{smallmatrix} (-4)5 \\ 8 \end{smallmatrix}$]	2.02758 942 [$\begin{smallmatrix} (-4)9 \\ 10 \end{smallmatrix}$]	2.43624 605 [$\begin{smallmatrix} (-3)2 \\ 10 \end{smallmatrix}$]	3.13130 133 [$\begin{smallmatrix} (-3)7 \\ \end{smallmatrix}$]	∞
5	1.13589 544	1.22344 604	1.31101 537	1.39859 928	1.48619 317	1.57379 213
15	1.14740 244	1.23727 471	1.32732 612	1.41751 762	1.50780 533	1.59814 200
25	1.17069 811	1.26548 460	1.36083 467	1.45663 012	1.55273 384	1.64899 522
35	1.20625 660	1.30915 104	1.41338 702	1.51870 347	1.62477 858	1.73124 518
45	1.25446 980	1.36971 948	1.48788 472	1.60847 673	1.73081 713	1.85407 468
55	1.31490 567	1.44840 433	1.58817 233	1.73347 444	1.88296 142	2.03471 531
65	1.38443 225	1.54409 676	1.71762 935	1.90483 674	2.10348 169	2.30878 680
75	1.45316 359	1.64683 711	1.87145 396	2.13389 514	2.43657 614	2.76806 315
85	1.49977 412	1.72372 395	2.00498 776	2.38364 709	2.94868 876	3.83174 200